GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION
SPONSORED PROJECT INITIATION

Date: December 16, 1977

Project Title: Finite-Element Analysis of Rapid Crack Propagation

Project No: E-23-634

Co-Project Directors: Dr. Wilton W. King and Dr. Jerry M. Anderson

Sponsor: National Science Foundation

Agreement Period: From 1/1/78 Until 6/30/80
(Grant Period -- Includes 6-month flexibility period)

Type Agreement: Grant No. ENG77-12948

Amount: $49,893 NSF
4,660 GIT (E-23-331)
$54,553 TOTAL

Reports Required: Annual Progress Report; Final Technical Report; Summary of Completed Project

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Defense Priority Rating: N/A

Assigned to: Engineering Science and Mechanics (School/Laboratory)

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OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Date: October 11, 1980

Project Title: Finite-Element Analysis of Rapid Crack Propagation

Project No: E-23-634

Project Director: Dr. Wilton W. King and Dr. Jerry M. Anderson

Sponsor: National Science Foundation

Effective Termination Date: 6/30/80 (Grant Period)

Clearance of Accounting Charges: - - -

Grant/Contract Closeout Actions Remaining:

- _ Final Invoice and Closing Documents
- X Final Fiscal ~~Report~~ Accounting (FCTR)
- X Final Report of Inventions
- _ Govt. Property Inventory & Related Certificate
- _ Classified Material Certificate
- _ Other __________

Assigned to: Engineering Science and Mechanics (School/Laboratory)

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CA-4 (1/79)

E-25-607

Annual Report (1/1/78-12/31/78): NSF No. ENG 77-12948

Finite-Element Analysis of Rapid Crack Propagation

Principal Investigators: W. W. King
J. M. Anderson

1. SUMMARY

A special finite element within which a crack tip can move continuously has been formulated. The geometry of the element facilitates its use with conventional elements modeling the remainder of a body. The crack-length dependence of the element's stiffness matrix has been identified as the mechanism for the release of mechanical energy as a simulated crack propagates in a discrete-parameter model. An established time-integration algorithm has been modified to accommodate the time (or crack length) dependent stiffness.

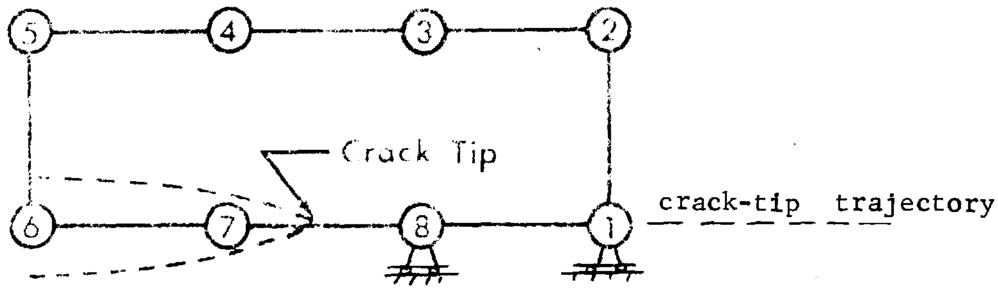
Numerical (computer) calculations have been carried out for an equilibrium problem of a center-cracked tension specimen to explore a potential scheme for ameliorating the incompatibility of displacements at interfaces of the crack-tip element with adjoining conventional elements. A Michigan State University experiment, involving the impingement of a stress wave on a stationary crack in a rectangular sheet, has been simulated, and the numerical results have been communicated to the experimentalists.

A closed-form approximation has been obtained for the stresses in a center-cracked tension specimen. This is expected to be useful in analyzing the initial motion of a crack accelerating from an equilibrium condition, since such a motion has a known analytical dependence on the crack-prolongation tractions prior to the motion.

2. RESEARCH ACTIVITY AND RESULTS

A. Element Geometry and Energy-Release Mechanism

The goal of this research is the development of a special finite element within which a crack tip can move continuously so as to allow numerical simulation of two-dimensional problems of rapid (unstable) opening-mode crack propagation. The basic geometry chosen for the element is shown in the sketch below.



The crack tip is allowed to take positions between nodes 7 and 8 so that the tip is always at least one nodal spacing from any boundary at which the element adjoins conventional elements representing the remainder of the body. The displacement field within the element is taken to be a linear combination of Williams' (or Rice's) eigenfunctions plus three rigid-body displacement modes; if we were interested only in the magnitude of crack extension permissible within the element, nodes 7 and 8 would be entirely superfluous. However, we envision the modeling of larger crack extensions by shifting the singularity element and restarting the internal propagation scheme after the crack tip has reached the position of node 8 in an element. Thus nodes 7 and 8 are desirable so that, after a regular nodal pattern for the structure has been established, the shift of a singularity element will not necessitate the sudden appearance and disappearance of displacement freedoms.

The displacement method of finite-element analysis has been adopted as the basis for our analysis in order that the strain and kinetic energies of the lumped-parameter finite-element model can be monitored in the numerical integration of the equations of motion. It is of course desirable that the finite-element model exhibit the absorption of mechanical energy that accompanies crack-extension and that can be put into one-to-one correspondence with the stress-intensity factor. For a convecting (with crack tip) assumed-displacement pattern Bazant, Glazik and Achenbach (Computers and Structures, 8, 1978) have derived, from Hamilton's principle, the form of the equation of motion in terms of nodal displacements

$$\underset{\sim}{M} \frac{d^2 \underline{u}}{dt^2} + \underset{\sim}{D} \frac{d\underline{u}}{dt} + \underset{\sim}{K}\, \underline{u} = \underline{F}(t)$$

where $\underset{\sim}{M}$, the mass matrix, $\underset{\sim}{D}$, an apparent damping matrix, and $\underset{\sim}{K}$, the stiffness matrix, are all time dependent in our case because of the fact that we use a spatially-fixed grid and the assumed displacement pattern moves within the special element which is surrounded by conventional finite elements. The time dependence of $\underset{\sim}{M}$ and the existence of $\underset{\sim}{D}$ follow from use of the consistent-mass allocation of inertia in the finite-element model. We have chosen, instead, to utilize the lumped-mass allocation of inertia which yields

$$\underset{\sim}{M} \frac{d^2 \underline{u}}{dt^2} + \underset{\sim}{K}\, \underline{u} = \underline{F}(t)$$

with $\underset{\sim}{K}$ time dependent and $\underset{\sim}{M}$ time independent. The energy-release mechanism is clear if we multiply by the transpose of the vector of nodal velocities,

$$\underline{\dot{u}}^T \underset{\sim}{M} \underline{\dot{u}} + \underline{\dot{u}}^T \underset{\sim}{K} \underline{u} = \underline{\dot{u}}^T \underline{F}$$

or

$$\frac{d}{dt} \left(\tfrac{1}{2}\, \underline{\dot{u}}^T \underset{\sim}{M} \underline{\dot{u}}\right) + \frac{d}{dt} \left(\tfrac{1}{2}\, \underline{u}^T \underset{\sim}{K} \underline{u}\right) = \underline{\dot{u}}^T \underline{F} + \tfrac{1}{2}\, \underline{u}^T \underset{\sim}{\dot{K}} \underline{u}$$

which states that the time rate of mechanical energy equals the rate of working of the external forces plus a term $(\tfrac{1}{2}\, \underline{u}^T \underset{\sim}{\dot{K}} \underline{u}^T)$ which, if negative, represents a dissipation of mechanical energy with increase in crack length. This is expected to be the case since the singularity element becomes "more flexible" as the crack length increases.

The "constant-average-acceleration" version of the Newmark-β algorithm for numerical integration in time has been adapted to the case of a time (or crack length) dependent stiffness matrix, and the essential computer programming will be completed during the coming summer.

B. Numerical Calculations for a Stationary Crack in a Stressed Sheet

Rather extensive numerical experiments have been carried out for the statics problem of a center-cracked sheet under tension and with the crack tip midway between nodes 7 and 8 on the singularity element. One objective was to determine the sensitivity of computed stress-intensity factors to the existence of node 7 which has the peculiar feature of lying on the free crack face where the displacement functions, used in the element, yield tractions that vanish identically. For each of the two problems considered stress-intensity factors computed with and without node 7 differed by less than 3%. The second objective was to explore the possibility of using fewer internal degrees of freedom in the element than displacement freedoms associated with nodes on the periphery; connections of the singularity element with adjoining structure was through a least-squares fit of nodal displacements to those evaluated from the internal displacement field. The ultimate purpose of such a modification would be a scheme to minimize the displacement incompatibility between the singular element and adjoining regular elements. The numerical results have exhibited an unexpected and significant sensitivity to the number of terms taken in the series.

A problem of stress-wave interaction with a crack in a rectangular sheet has been modeled. The numerical analysis simulates an experiment conducted at Michigan State University by Prof. W. N. Sharpe, Jr. and a graduate student. The finite-element results have been communicated to Prof. Sharpe for comparison with the experiment.

C. Analytical Estimate of Effect of Finite Width on Stresses in a Center-Cracked Tension Specimen

A closed-form approximation obtained for the effect of finite-width on stresses in a center-cracked tension specimen has been motivated by (a) the need for analytical benchmarks against which to compare numerical analyses and (b) the fact that the initial motion of a crack accelerating from an equilibrium condition has a known analytical dependence on the tractions on the crack's prolongation prior to the motion.

By combining the classical infinite-sheet solution with other acceptable singular solutions of no inherent practical interpretation, one arrives at a stress distribution that satisfies all the appropriate field equations and balances the tensile resultant transmitted across the ligaments with that of the remotely applied stress (σ) without the imposition of a periodic deformation pattern to the sides of the strip. The resulting formula giving stress-intensity factor (K_1) as a function of width-normalized crack length ($\lambda = a/w$) is

$$K_1 = \sigma\sqrt{\pi a}\left(1 + \frac{1-\sqrt{1-\lambda^2}}{\lambda \log \frac{1+\sqrt{1-\lambda^2}}{\lambda}}\right)$$

Isida's truncated series of eight terms which ultimately raise λ to the fourteenth power is regarded as the standard of comparison for width-effect estimates-even though it fails to provide the proper asymptotic behavior as $\lambda \to 1$. The formula given above agrees with Isida's results to within 4% in the range $0 \leq \lambda \leq 0.9$ and moreover satisfies the limiting requirement that $K_1 \to \infty$ as $\lambda \to 1$.

3. MOST SIGNIFICANT RESEARCH ACCOMPLISHMENTS

(a) Identification of Energy-Release Mechanism in Finite-Element Model

(b) Analytical Estimate of Finite-Width Effect on Stresses in Center-Cracked Tension Specimen.

4. PERSONNEL SUPPORTED THROUGH THIS RESEARCH

(a) Prof. J. M. Anderson: analytical estimate of finite-width effects; finite-element modeling.

(b) Prof. W. W. King: singularity-element development and selection of time-integration algorithm

(c) Prof. R. W. Shreeves: assistance with computer system and program diagnostics

(d) S. G. Moran graduate assistant: computer programming; execution of sample problems

5. TECHNICAL PAPERS

"Derived Finite-Width Corrections in Fracture Mechanics" in preparation for International Journal of Fracture.

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

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PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Institute of Technology Atlanta, Georgia 30332	Solid Mechanics	ENG 77-12948
	4. Award Period From 1/1/78 To 6/30/80	5. Cumulative Award Amount $49.893

6. Project Title

Finite-Element Analysis of Rapid Crack Propagation

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

For two-dimensional opening-mode linear elastic fracture mechanics, a special finite element, within which a crack tip can move continuously, has been formulated. Generalized coordinates, one being the stress-intensity factor, are associated with crack-tip eigenfunctions so that the stress field within the element exhibits the appropriate asymptotic form. The crack-length dependence of the element's stiffness matrix is the mechanism for the release of mechanical energy as a simulated crack propagates in a discrete-parameter (finite element) model, and numerical calculations have shown this energy-release rate to be consistent with the directly determined stress-intensity factor for unstable propagation at realistic speeds. In test cases the performance of the finite-element analysis has been excellent, provided the crack extension is confined to one element; the process of shifting elements in the network to accommodate greater extension has induced spurious oscillations in structural response. Several attempts to ameliorate this problem have been unsuccessful.

A byproduct of this investigation has been the development of a closed-form approximation for the stresses in a center-cracked tension specimen. This is expected to be useful in analyzing the initial motion of a crack accelerating from an equilibrium condition, since such a motion has a known analytical dependence on the crack prolongation tractions prior to the motion.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (✓)	Approx. Date
a. Abstracts of Theses		x			
b. Publication Citations		x			
c. Data on Scientific Collaborators		x			
d. Information on Inventions	x				
e. Technical Description of Project and Results				x	11/30/80
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
W. W. King; J. M. Anderson		9/25/80

NSF Form 98A (5-78) Supersedes All Previous Editions

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III. 1. a.

Some Modifications of a Special Finite Element for Two-Dimensional Linear Fracture Mechanics

by Stephen G. Moran

In Partial Fulfillment of the Requirements for the Degree Master of Science in the School of Engineering Science and Mechanics, Georgia Institute of Technology, August 1979.

Abstract

A special crack-tip finite element has been modified so as to

1. allow a reduction of generalized coordinates with attachment to surrounding elements by continuity of displacements at nodes in a least squares sense, and

2. accommodate a variable crack-tip location within the element.

Generalized coordinates for the cracked element are associated with the displacement eigenfunctions for the plane-strain opening-mode behavior of a straight crack. Displacement continuity can be maintained at boundary nodes and those nodal displacements will constitute independent degrees of freedom provided the number of element generalized coordinates matches the number of nodal displacements. With fewer generalized coordinates, either the independence of nodal displacements or continuity of displacements at boundary nodes must be sacrificed; here the latter approach was taken, with continuity maintained in a least squares sense. The motivations were economy in the development of a variable-crack-position stiffness matrix and elimination of undesirable high frequency natural modes of the element. Results of calculations for test cases demonstrated that accuracy of computed stress-intensity factors was too rapidly degraded with a slight reduction in generalized coordinates for the approach to be useful.

Three methods of numerical integration were used to generate a crack-length-dependent stiffness matrix for the crack-tip element. For accuracy and economy Gauss-Legendre quadrature was found to be superior to both Simpson's rule and the trapezoid rule.

III. 1. b. Publications

1. "Fast Fracture Simulated by Conventional Finite Elements: A Comparison of Two Energy-Release Algorithms," Crack Arrest Methodology and Applications, American Society for Testing and Materials, STP 711, 1980, J. F. Malluck and W. W. King.

2. "Yet Another Finite-Width Correction for the Center-Cracked Tension Specimen," submitted to International Journal of Fracture, J. M. Anderson.

3. "Finite-Element Simulation of Unstable Crack Extension with Singularity Elements," planned to be submitted to International Journal of Fracture, W. W. King, D. Shaw and J. F. Malluck.

III. 1. c. Scientific Collaborators

R. W. Shreeves, Associate Professor
J. F. Malluck, Senior Aircraft Structures Engineer, Lockheed-Georgia Company
S. G. Moran, Graduate Student
Dein Shaw, Graduate Student